REVIEWING TARSKI's SEMANTIC and MODEL CONCEPT. 85 years later ... *

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Abstract

In trying to formalize the theory of elaborating and using simulation games supported by a computer and only using natural everyday language expressions raised many fundamental questions leading back to the roots of logic and semantic theories. Tarski has contributed some important papers. Two of them are discussed in this text with the mentioned project in mind.

1 85 Years Later

The two papers of Tarski, which I do discuss here, have been published in 1936.[1] Occasionally I have already read these paper many years ago but at that time I could not really work with these papers. Formally they seemed to be 'correct', but in the light of my 'intuition' the message appeared to me somehow 'weird', not really in conformance with my experience of how knowledge and language are working in the real world. But at that time I was not able to explain my intuition to myself sufficiently. Nevertheless, I kept these papers – and some more texts of Tarski – in my bookshelves for an unknown future when my understanding would eventually change...

This happened the last days.

[1]See Tarski (1936) [Tar36a] and [Tar36b].

2 The Simulation-Game Case

As the reader can verify, in this uffmm.org blog I try to apply some example of an *integrated engineering theory* called DAAI (distributed Actor-Actor Interaction) further developed into a paradigm called ACA (Applied Cultural Anthropology) to an application case where a group of people wants to share their own knowledge in a way, that (i) they can use their shared knowledge as a simulation game with themselves as players. Furthermore (ii) this simulation should be able to be supported by a computer doing all the book-keeping work, and finally (iii) there should be no special programming at all; the natural language which the group is talking should be enough. Doing this the computer becomes an invisible assistant thinking exactly what they are thinking.

While I was elaborating the requirements for our software which shall support our vision, I detected many questions (this is normal :-)) but I found also some interesting answers (this is nice, when it happens :-)). If you would take a short look to this small text[2] You can see, that I have used the concept of *satisfiability* of an expression with regard to a *model*, but I have used these concepts according to the general manner how it is done in mathematical model theory. Doing this I already detected that the schema of model theory in general was applicable in my case but strictly speaking this was beyond the boundaries which are accepted in mathematical model theory. The main difference results from the fact that mathematical model theory is using some *mathematical* structure to set up a model, but in my case I had real actors with some *built-in meaning functions* π which are using their meaning functions and the structures determined by their meaning functions as models. These structures can be mathematical structures, but usually these structures can be *some other structures too*. Mathematical structures are only one kind of structures.

At this point I was reminded at *Tarski* and his foundational considerations about semantics, satisfiability of expressions with variables and models. Rereading his papers again in this situation was really enlightening.

3 Tarski about 'Satisfiability'

The first paper I was rereading was his famous paper *Über den Begriff der logischen Folgerung*.[3] After some introductory remarks about logical consequences he introduces the idea of satisfiability and a model.

[2]See https://www.uffmm.org/2020/07/26/komega-requirements-no-1-basic-application-scenario/

[3]See the English version of this paper in Tarski (1956)[Tar]:Chapter XVI, *On the concept of logical consequence*.

He presents the general idea with *logical expressions* whose *constants* can be replaced by *variables* and that there can be objects which can *satisfy* the expressions by showing that they are possible instances of the variables. The examples which he presents are in the German text as follows:

1. Schema: *X and Y are brothers*. There are two persons named 'Johann and Peter' which are assumed to be brothers. After replacing the variables 'X' and 'Y' by the constants 'Johann' and 'Peter' the schema is said to be still valid, which is interpreted by Tarski in the sens that these constant *satisfy* the schema.

2. Schema: $x + y = z$. There are the three numbers '3','2', and '5' which – if one replaces the variables 'x', 'y', and 'z' by these constants – do *satisfy* the schema according to Tarski.

Although Tarski mentions that a general definition of this satisfiability concept depends in some sens from the specialties of the language in use he thinks that these examples can be sufficient to get the idea of the general concept.

General Model of Satisfiability: While Tarski proceeds in his text immediately with logic only I like to point out, that a *general schema* of *satisfiability* seems to include the following elements:

1. One needs an *expression* e of some *language* L where the expression can have 'parts of speech' which can either be *variables* or *constants*. The constants are understood as *possible instances* for these variables.

2. The schema represents some kind of a *relation between* the variables.

3. To decide whether a constant can *replace/substitute* a variable or not one needs some *criterion/ rule*.

There are at least two different views possible about this concept of satisfiability: (i) A *purely formal* approach defining only some sets of expressions and rules how to arrange such expressions and how to substitute some expressions by others. (ii) A formal approach *extended by some semantics* by mapping the expressions into some situations beyond the expressions themselves having objects, properties, relations, and some dynamics which are behaving according to their *own rules*. The constants are in this case names of elements from this situation. Replacing variables by constants which are embedded in some *lawful structure by their own* requires for the judgment of satisfiability to compare the *relations of the schema* with the *relations of the selected situation*. This can not be done in a purely automatic way. It requires to check every relation of the schema against the different relations of the selected situation. Thus talking about *satisfiability* in the *semantically enriched schema* calls for *special knowledge* for those which are interacting with a special situation.

This last case is given in the case of *natural language communication*. With regard to the subject of 'satisfiability' every natural language includes at least the following components: (i) The senses of the body generate for the brain complex neural events which can be viewed as individual, concrete *percepts* which from the brain will automatically be processed as *instances* of more *general concepts* already been built up by the learning history with past percepts. Language expressions are always using names of such general concepts which function here as a kind of *material variables* representing a vast collection of percepts (= constants) which are acknowledged instances of the general concept (= variable). Thus the expression 'table' is a material variable for many concrete percepts (possible instances) of the general concept. The same with 'cup', 'tree' etc. Thus the everyday language is using the concept of satisfiability as a *normal case* to be able to talk with a small number of expressions about a huge amount of possible instances. The *criterion*, the *rule* by which it will be decided whether a percept is an allowed instance of a general concept is located in the *individually learned meaning function* π of *every speaker-hearer* of the language. These general concepts are not given by nature, not given a priori, but have to be learned step by step in an inductive way. Because the perceived *real world* RW is *given* for human speaker-hearers, the way *how* a certain population *names* these perceptions is usually completely different.

Normal, everyday language is using the schema of *satisfiability* in many different ways. One well known schema is the way how *structures of sentences* are organized. Pointing to the simple schema 'S-P-Ext' (Subject - Predicate - Extension') allows the construction of many expressions like *Peter* | *eats* | *some food*, *The car* | *is driving* | *too fast*, *Guns* | *are calling* | *for enemies*, etc. Thus natural language is a great master in using the schema of satisfiability, and these schemes are all *highly dynamic*, because the *set of possible instances is never closed*!

4 Tarski about 'Model'

Having introduced the concept *satisfiability* Tarski is introducing the concept of *models*.[4] He is now calling the before introduced concept of *instances* of *variables* a *model*: all the instances together which are *satisfying* the variables of a schema are a *model* for this schema.

[4] In the German text Tarski uses the plural of the word 'model'!

Although Tarski is using this concept of model only in the case of logical expressions one can apply this concept – as discussed before – in the case of a normal language too. To speak about the 'trees' of your 'garden' is usually understood as being *true* as long as there are some concrete real things in your garden whose percepts are matching the general concept of 'trees'. Then these tee-like percepts are a possible instance satisfying the concept and therefore they function as a model for the expression. The same holds for the general concept of 'garden'. Thus *normal language expressions* are using the *concept of a model* nearly as a *standard case*.

5 Tarski about 'Logical Consequence'

Before the concept of a 'model' was available logicians had already a concept of *logical consequence* $\vdash$, which related a set of expressions K assumed to be 'true' and some new expression ϕ, from which it had to be 'shown' that it is true too, written as

$$K \quad \vdash \quad \phi \tag{1}$$

A so-called '*proof*' that the expression ϕ is '*logically true*' is a process called '*deduction*' where a finite 'set of *inference rules*' Ξ describes, what can be done to generate a finite sequence of expressions $\langle e_1, e_2, ..., e_n = \phi \rangle$ where the *last* expression e_n is that expression ϕ which shall be declared as being *logically true* based on this sequence of expressions *derived* from the set K only by *applying defined inference rules* from Ξ.

As known from the history of logic[5] the definition of *purely syntactical inference rules* is cumbersome and requires a kind of *intuition* which is only *implicit* and not completely in accordance with everyday intuition. Finally the intuition helpful for the construction of the formal machinery is abandoned as soon as the mechanism 'works'.

With the concept *model* at hand one can *substitute* the syntactical concept of logical consequence by a *model-based* concept, written as:

$$M \quad \models \quad \phi \tag{2}$$

A *proof* by a model states that the *model* M *satisfies* the expression ϕ or the expression ϕ is satisfied by the *model* M. This implies that the model M is also a model for the set of expressions K:

[5] See e.g. Kneale & Kneale (1962)[KK62].

$$K \vdash \phi \quad iff \quad \forall(M)(M \models K \Leftrightarrow M \models \phi) \tag{3}$$

This equivalence of the *syntactical* logical concept of consequence and the *model-based* one can be exploited in many different ways. Tarski – and nearly all logicians and mathematicians after him – have used this equivalence to argue that the derivation of *true expressions* from a *set of already assumed true expressions* can be done without any kind of *meaning beyond the formalism*; such a concept like 'meaning' is only *cluttering* the clear formal concepts by an uncontrollable fuzziness due to implicit intuitions.

This position has some truth on its side, but not all :-) If one works with formal structures and formal derivations in the real world there comes the point where you want to use these formal structures to describe some (complex) phenomena in the real world.[6] In that moment of *applying formal structures* to *reality* one has to establish some kind of an *interpretation*, a *mapping* between those parts S of the real world RW which one wants to describe and those formal expressions K which one wants to use for such a description:

$$S \subset RW \tag{4}$$
$$\pi_{rw} : S \longleftrightarrow K \tag{5}$$

Formal logic as such has no device to install such an interpretation. Formal logic lives by definition by its formal apparatus only. But now, after the introduction of the concept of a *model* one can be inclined to apply this concept in this special case. The basic idea could be:

$$S \subset RW \tag{6}$$
$$\pi_{rw2} : S \models K \tag{7}$$

Intuitively one interprets the subset S of the real world RW as a 'model' which satisfies the set of formal expressions. In the case of natural language expressions $K \subseteq L_0$ this idea is the 'standard case' because every speaker-hearer has its *built-in meaning function* π which relates the neural correlates S_{nn} of the perceived real-world subset S with the neural correlates K_{nn} of the real-world formal expressions K as soon as he – together with others – has established such a relation. Human persons can do this all the time, very easily. But the *model-concept* introduced by Tarski has no kinds of actors with built-in meaning functions. There is no device around which could do the job. The Tarski-model-concept is - like formal logic in general – a purely formal concept

[6] An example is the case of applied physics.

of relating one kind of expression – the model M – with some other expressions – the expressions K which shall be satisfied –. While in the case of the *syntactical* logical inference I have a formal description how to 'play' with other formal expressions to construct a sequence of formal expressions, in the *model-based* logical inference I have no formal description how to construct a model which satisfies formal expressions.

6 Models and the 'Meaning Device' α

Let us go back to the starting point of these considerations, the case of simulation-games where a group of people wants to describe their own knowledge in a way that it can be used as a model for a simulation game. The idea is that these people do not use a special language like the *language of formal logic* $L_{\vdash}$, which has no device to relate this language to the real world, but they are using the *normal, everyday language* L_0 because this language has a *meaning function* π which is (i) *part of the language concept* and (ii) is *built-in* in every speaker-hearer of a natural language.[7] Thus whichever expressions a natural language is using[8] the speaker-hearer have always many interpretations at hand.

Therefore it makes not too much sense to abandon natural language only by the argument, that it is *difficult* to clarify the implicit meanings with a 100% clearness. In case of formal languages we have *no* meaning at all and it is not possible to construct *any kind* of meaning using only formal systems! The empirical sciences are trying to use – partially – formal systems for their descriptions, but exactly these applications are until today *not really solved*.[9]

That device which shall *bridge* the *gulf* between formal expressions and the real world is in empirical sciences called *measurement*. A measurement is a process in the real world which *translates/ maps* special real-world events or properties P_{rw} into some artificial *data* which have a defined *symbolical/ formal expression* as a 'name'. The measuring process includes always a *comparison operation* between the real-world object P_{rw} *to be measured* and a real-world object ST_x as the *standard* to compare with. An example is the measuring

[7] Biology tells us that human persons classified as the life-form called *homo sapiens* is the only known system in the whole known universe which possesses – besides many other capabilities – the capability to learn and use symbolic languages 'by nature'. This capability is 'built-in' which is due to a development time of 3.5 Trillion (10^{12}) years. Because this capability enabled cooperation between humans in a new dimension allowing culture and technology, one can – or even 'must' – say that humans are *language animals* having the power to change the whole universe!

[8] Think of English, Russian, Arabic, Chinese, ...classical Hebrew, classical Greek, classical Latin, Sanskrit, ...

[9] See e.g. Suppe (1979)[Sup79].

of the *length* of a real-world object according to an *agreed standard* like the *meter [m] unit* is . This comparison can yield perhaps a *data expression* like *4.5 m*. The expression '4.5 m' as such has no meaning. But in a relation like $M(O_{rw}, ST_m, '4.5', 'm')$ with 'M' for measurement it would in principle be possible to relate the expression '4.5 m' to the two real world objects 'Meter Standard Object' ST_m and 'Object-to-be-measured' P_{rw}. But because the 'expression as such' is only an expression without any meaning such a measurement procedure as such has no meaning too. Connecting measurement-devices with other devices which completely *automatically* measure real-world properties and process them purely automatically[10] one establishes a real-world process with *chained real causal links* where meaning does not matter too.

In the real world *Meaning* is the value of a *mapping* which is by definition not a fixed causal link but a flexible correlation between two different things happening primarily in the *realm of neural states* of an speaker-hearer which can *change all the time*. This is bad if one wants *stability*, but it is *luck* if the surrounding world is steadily changing and you are in need to adapt your inner states according to the changing situation. As long as the properties of the surrounding real world (including the body!) are stable one can keep the meaning relations stable as well (like a measuring standard). As soon as the properties of the real world are changing one can –and should – change the meaning relations accordingly.

Thus there is no direct way from formal expressions E_{rw} to some parts of the real world P_{rw} without relying on an adaptive input-output system α, which can establish *meaning functions* by (i) transforming real world properties P_{rw} into correlated inner states $IS_{\alpha.P}$ as well as real world expressions E_{rw} into correlated inner states $IS_{\alpha.E}$ and then (ii) establish a mapping on both sets of inner states $\pi_{alpha} : IS_{\alpha.E} \longleftrightarrow IS_{\alpha.P}$. Clearly there must exist a further mapping from the inner states of the system IS_{α} back into the real world properties. Thus we get the following minimal requirements for the generation and usage of *meaning* in our known world:

Def. Dynamic Meaning Device α:

$$P_{rw}, E_{rw} \subseteq RW \tag{8}$$

$$\alpha_{perc} : RW \longmapsto IS_{nn} \tag{9}$$

$$IS_{nn.P} \cup IS_{nn.E} \subseteq IS_{nn} \tag{10}$$

$$\pi_{\alpha} : IS_{nn.P} \longleftrightarrow IS_{nn.E} \tag{11}$$

$$\alpha_{com} : IS_{nn.E} \cup IS_{nn.P} \longmapsto E_{rw} \tag{12}$$

[10]For instance measuring temperature and the dryness of soil could trigger automatically spreading of some water through pipelines.

In the last expression $\alpha_{com} : IS_{nn.E} \cup IS_{nn.P} \longmapsto E_{rw}$ it is told that expressions in the real world can also talk about *other expressions*. In formal languages this is called the case of a *meta-language* talking about another language as its *object-language*. While in the realm of formal languages this can be tricky and can pose many problems natural languages can use many levels of languages at the same time and with ease. The meaning function *built into* a human speaker-hearer has *by nature* all these capabilities. This is great! The often mentioned *logical paradoxes* which can be produced in formal languages as well as in natural languages are in the case of natural languages not a *defect* but show only, that the power of natural language can produce logical problems if *used in a wrong way*! The natural language does not urge any speaker-hearer to do this. You are always allowed to *use it in the right way*!

7 What can be Done?

Which consequences can we draw from these considerations? Actually we have differentiated the following cases:

$$K \vdash \phi \quad (13)$$

$$M \models \phi \quad (14)$$

$$\alpha : M_{rw} \longmapsto L_0 \quad (15)$$

$$m \in M_{rw}, \phi \in L_0 :$$

$$\alpha(m) =_m \phi \quad (16)$$

The classical formal logic consequence $\vdash$, the *model-based* satisfiability $\models$ and an input-output based meaning relation $=_m$.

In the simulation-game case we have two different sets of expressions: (i) the set Z of an actual static state where it is assumed that all expressions correspond with matters in the real world and therefore are assumed to be *true* by the experts, (ii) the set X of expressions having in their *condition part* X_{if} expressions which describe matters in the actual state which should be directly be *decidable* whether they *do occur (= being true) or not*. This case allows the following readings:

1. $Z \vdash X_{if}$: It is possible by formal rules to prove that X_{if} is true.
2. $Z \models X_{if}$: It is possible to show that the state Z satisfies the expressions in X_{if}.
3. $alpha(Z) = Z_{rw} \otimes \alpha(X_{if}) = X_{if.rw}) \otimes \alpha : Z_{rw} \models X_{if.rw}$: An input-output meaning device α can translate the expressions of Z and X_{if} in the meaning correlates and can then decide whether the meaning correlates

Z_{rw} of Z do satisfy the meaning correlates of $X_{if.rw}$ of the conditions X_{if}.

Although we are planning to support the human experts by a computer it has to be acknowledged that the *full case* of an input-output meaning device α will not be feasible in the near future. But it is an interesting question whether there exists a *sub-case* $\alpha*$ of an input-output meaning device α which can handle a sub-language $L_{0.0} \subset L_0$ in a sufficient way.

To define such a sub-language $L_{0.0}$ it can be of help to have a minimal requirement. Such a minimal requirement can be given by remembering another paper of Tarski from 1936 [Tar36a] with the title *Grundlegung der wissenschaftlichen Semantik*[11] Talking about the general idea of the *truth of a sentence* which Tarski is locating in the correspondence of the sentence with reality (cf.EN: p.404) he proposes a meta-language statement as a general description of this. He states: "*the sentence x is true if and only if p*. And he comments this statement by saying that *'p'* is a variable which can be replaced by any sentence of the language L under investigation and *'x'* can be replaced by any individual name of the sentence used for p.(cf. EN p.404).

He presented some examples to illustrate his general schema like the following one: the sentence *'it is snowing'* is true if and only if *it is snowing*.

The idea is, that the sentence *it is snowing* is an expression pointing to some *meaning* which is hided behind the inner meaning function ϕ of the speaker-hearer and the expression *'it is snowing'* is a *name* of this sentence therefore pointing to the sentence with the hidden meaning. As discussed above this way to deal with the meaning of natural language is completely unsatisfying because it explains nothing about meaning and therefore it does not explain the idea of correspondence between a natural language expression like *'it is snowing'* and its possible meaning reaching out *beyond the expression itself*.

Nevertheless even this *truncated idea of correspondence* can be of help for our task, because the mechanism that the conditional part X_{if} of a change rule includes different names of different expressions in Z would allow a first simple mechanism: if e.g. in Z there exists an expression like *Peter sits at the table* and in X we would have as a condition *'Peter sits at the table'* then one could immediately identify that the expression of Z satisfies the name of X_{if}. Depending from such a satisfaction an effect could be triggered.

Starting from such a first general rule one can proceed and collect more decidable conditions for satisfiability. Another example could be that one makes

[11] See the English version in Tarski (1956) [Tar] *Establishment of Scientific Semantics*.

use of the *general concepts* of natural language like *some man*, *some woman* or *some animal* or *some robot* which functions like a *variable* which can be replaced by agreed *constants* which could be elements of a list of allowed constants for such a variable.

Furthermore there exists the general assumption of a *world clock* starting with the first state. One could use agreed wordings to refer to a certain point in time or to certain time spans like: *the last x minutes*, *the next x minutes*, *the next days*, *after 5 years*, *in 10 yeras*, etc.

There is the other general assumption about an assumed 3D-space as a general framework. All objects mentioned in a state have in principle a *location* in this space, even if this has not been stated explicitly. Such a location can be given with *coordinates* or *sets of coordinates* or *named regions* or relatively like *besides*, *on*, *under* and the like.

As next step there should be run several experiments in testing which kinds of states and changes can be described with such a *minimal language* $L_{0.0}$.

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